Voluntary Pooled Public Knowledge Goods and Coalition Formation

T. Dedeurwaerdere and P. Melindi Ghidi

Discussion Paper 2013-20

Institut de Recherches Économiques et Sociales
de l'Université catholique de Louvain

UCL

Voluntary Pooled Public Knowledge Goods and Coalition Formation

Tom Dedeurwaerdere*and Paolo Melindi Ghidi†

Abstract

In this paper we develop a theoretical model of the mechanisms behind the voluntary provision of public knowledge goods in coalitions in presence of social preferences. The model builds on the large empirical literature on voluntary production of pooled public knowledge goods, such as source code in communities of software developers or data voluntarily provided to open access data repositories. This literature shows that the provision of public goods is strongly dependent on the presence of social preferences such as group identity and social approval of individual pro-social attitudes. To integrate these effects in standard public good theory this paper builds a private-collective model of public good provision, where contribution to public knowledge goods generates both public and exclusive private benefits for the members of the coalition only. The analysis shows that, when the private benefit is important, the effect of the social preferences on the coalition formation is ambiguous. In particular, in the latter case, in presence of strong individual reputational effects, the public knowledge goods will be more difficult to produce. The comparison of the predictions of the theoretical model with the stylized facts of large scale surveys of Free/Libre/Open-Source (FLOSS) software developers confirms the results of the model.

Key words: coalition formation, private-collective model, social group identity, pro-social reputation, public knowledge goods, social dilemma.

JEL Codes: H42, D71

*Université Catholique de Louvain and Fonds National de la Recherche Scientifique (FSR-FNRS)

†Université Catholique de Louvain

1 Introduction

A vast body of empirical research has shown the effectiveness of initiatives for the production of public knowledge goods through voluntary mechanisms (Lessig 2001, Benkler 2006, Hess and Ostrom 2007, David 2008). From the point of view of public good theory, two important features of successful economic and institutional arrangements are highlighted in this literature: first, the importance of reaching an adequate group size for the knowledge generation process to proceed and, second, the effect of private benefits on the willingness of agents to participate to the public knowledge production through coalitions.

The first feature has been widely analyzed in empirical surveys of open source software communities. For example in a recent large-scale survey, Schweick (2012) has shown that reaching a certain group size is a condition for developers to effectively pool their efforts and contributions. In other terms, only if the aggregate level of produced public good is high enough, agents will be motivated to voluntary join the pool. In a model of open source software, Johnson (2002) shows that increasing the development of open source applications with a modular structure depends on the reaching of a critical size of the developers base.

Further, when the minimal coalition size is reached, this size will theoretically also be the optimal size as there is no economic incentive for other agents to join the contributors group as they can free ride upon the public good once it is produced. This duality between the group of user-developers on the one hand and the broader user group on the other is a well-known pattern that is observed for instance in open source development projects (cf. for example Raymond, 1999, p. 215). Indeed, in these projects, the group of users-developers grows till the group-size is sufficient for the task at hand,

while the broader user group of the software tool continues to grow beyond that size.

The second feature, which can also be illustrated by examples from the empirical literature on open source development, is that voluntarily provided knowledge goods are public goods with a joint public/private character (von Hippel and Krogh, 2003). This means that these goods generate both aggregate public and personal private benefits to the contributors, such as private problem solving, learning and enjoyment, higher citations for researchers through increased visibility, or access to new personal competences by joining a group with high-level expertise. As a consequence, agents contributing to public knowledge pools are both driven by the public good benefits and the private benefits generated by the production of that good. Evidence for this mixed public/private character has been given in many studies of public knowledge goods in fields beyond the case of the software developers communities. One can think for example of open access databases with tailor made data management tools that benefit specific communities and individuals (David, 2005) or hybrid funding arrangements, including both market and non-market tools, for openly available culture products on the Internet (Lessig, 2008).

As has been shown elsewhere, the joint public/private character of these particular type of goods can lead to contrasted effects on public good production. In particular, if the private benefit is a substitute to an equivalent private benefit that can be otherwise obtained through the purchasing of a private good on the market, the joint character might lead to a decrease in the willingness to contribute to the public good in situations where the market price of the substitute is sufficiently low (Cornes and Sandler, 1984). Two major issues are however not addressed in the existing theoretical models: first, to what extent is this ambiguous effect attenuated or exacerbated by coali-

tion formation? Second, what is the role of group related social preferences in such coalitions on the willingness to contribute, such as group approval for individual pro-social behavior or social identity benefits from group belonging?

To answer these questions, this paper develops a theoretical model of the mechanisms behind the production of public knowledge goods through coalitions, with the view to analyze these ambiguous effects. To the best of our knowledge, this paper is one of the first attempts to build a theory of public knowledge goods in the context of coalition theory.

To build the model of coalitions with social preferences, this paper proceeds in two steps. The first step builds upon the public good model with institution formation of Kosfeld et al., (2009). To extend this model to the case of pooled knowledge goods with exclusive private benefits to the coalition members, this paper draws upon the private-collective incentive theory developed by von Hippel and von Krogh (2003). The second step builds upon the theory of intrinsic motivation (Benabou and Tirole, 2003) and applies this theory to group behavior based on the models from the vast social psychological literature that analyses the role of social preferences in extra-role behavior and organizational citizenship behavior (Lepine et al., 2002).

The main results of the paper are the following. In the model with standard preferences, the private benefits that are tied to the production of the public knowledge good makes coalition formation easier, as it is sufficient to reach a smaller coalition size in presence of private benefits. Adding social preferences for group belonging and individual pro-social behavior introduces a new effect on the coalition size. Groups that reward social group identity related to the production of the collective good, such as social programmers in open source software communities, will make coalition formation easier. In contrast, groups that reward individual pro-social behavior will make coalition

formation more difficult when the private benefits deriving from the production of public knowledge goods in a coalition are sufficiently large, such as hackers in open source software communities which are driven by individual problem solving and personal career benefits. This new effect in turn leads to an alternative strategy to overcome social dilemmas. Indeed, application of the theory to a population composed of two groups with different levels of social preferences shows that groups with homogeneous social preferences and small private benefits can overcome the social dilemma by broadening the homogeneous group to a larger heterogeneous group. A comparison of the predictions of the theoretical model with the stylized facts of large scale surveys of Free/Libre/Open-Source Software (FLOSS) developers confirms the results of the model.

The paper is organized as follows. Section 2 provides the basic motivation and facts of our analysis of production of public knowledge goods and presents some stylised facts. Section 3 theoretically analyses the analytical conditions under which social dilemmas arise and how these can be overcome through coalition formation in a private-collective model with standard preferences. Section 4 introduces social preferences and the concept of social network coalitions. In Section 5 we present an application of the model assuming heterogeneous preferences. Section 6 concludes.

2 Motivation and Facts

The contrasted effects of social preferences on the size of coalitions in coalition formation processes can be illustrated empirically by an in depth analysis of the motivations, personal attributes and behavioral patterns among free/libre and open-source (FLOSS) developers, based on the FLOSS-US 2003 survey (David

and Shapiro, 2008). This web-based survey generated a wealth of data on motivations and reasons for developers to begin to work for FLOSS. Using this data, the study by David and Shapiro classifies the respondents according to their distinct motivational profiles by hierarchical cluster analysis. In addition, whenever possible, the respondents in each cluster are also matched to projects of known membership sizes, revealing that the fractions of respondents from each motivational cluster for the large and the very small project ranges are statistically different, as shown in table 1.a and 1.b. below.

Two major outcomes from this study cannot readily be explained in a model with standard preferences alone. The first feature is related to the effect of social preferences on coalition formation and the contrasting effects of these social preferences in case of low versus high private return for the members of the coalition. As can be seen from this study, the three clusters of social learners, social programmers and user-innovators are present both in the small and large ranges of the project sizes, indicating that for these groups it is easier to form coalitions, compared with other clusters that are only present in the large ranges. This is consistent with the fact that group belonging and social group identity related to collective good provision fosters cooperation in situations of social dilemma. In contrast, the cluster of aspiring hackers, which is composed of individualist, materially motivated programmers, which take part to FLOSS in the interest of a future career (David and Shapiro, 2008, p. 383), is by far more present in the large-size groups than in the small-size groups. This is consistent with the fact that, in presence of large private returns of the collective goods, and in the absence of strong social group identity related social preferences, coalition formation is comparatively more difficult, as private returns decrease the social approval effects. In such case the willingness to join a coalition will depend to a larger extent on the

Table 1: Motivational Clusters among Open Source Software Developers

Cluster	Profile	Key characteristics
1	Professionals	Non-ideological, expert, self-employed or company-sponsored to collaborate on FLOSS projects
2	Aspiring hackers	No need to modify existing code but like fixing bugs and learning new programs
3	Social learners	Become better programmers, learn how programs work, work with like-minded, "give back to community," support FLOSS ideology
4	Social programmers	Experienced, employment related needs to use, modify existing code and fix bugs; project choice influenced by social connections with other developers
5	"User-innovators"	Modifying existing software unimportant, learning and interacting with like-minded others unimportant; wanted to "give back to community," and launched own project

(a) Key characteristics of motivational clusters (source : David and Shapiro, 2008 : p. 384.

Cluster		Small project and large project populatons only		
		Small (1–2)	Large (>29)	Total
1 (Professionals)	%	**5.2**	**5.5**	**5.3**
	N	22	10	32
2 (Aspiring hackers)	%	**7.6**	**16.7**	**10.3**
	N	32	30	62
3 (Social learners)	%	**49.1**	**45.0**	**47.8**
	N	207	81	288
4 (Social programmers)	%	**14.0**	**12.8**	**13.6**
	N	59	23	82
5 (User-innovators)	%	**24.2**	**20.0**	**22.9**
	N	102	36	138
Total	%	**100.0**	**100.0**	**100.0**
	N	422	180	602
Pearson chi-squared(4)			11.09	
Prob > chi-squared			***0.03***	
Chi-squared goodness-of-fit(4)			60.75	
Prob > chi-squared			***0.00***	

(b) Distribution of small and large project participants by motivation profiles identified by cluster analysis of FLOSS-US survey respondent (source: David and Shapiro 2008, table 12, p. 394.

marginal return on the aggregate public good that one only obtains in larger groups.

The second feature is related to the contribution of heterogeneous coalitions to overcome social dilemmas when homogeneous groups are too small to form viable coalitions. Table 2 gives the matrix for developers movements among projects of different membership size. In reading these data, it is reasonable to make the hypothesis that on average developers in their first project derive higher personal learning and problem solving benefits (higher private benefits), compared to the involvement of these same persons in their second and/or most recent project, especially if they were involved in their first project in projects of small and medium size. Indeed as stated by David and Shapiro (2008, p. 390) while these communities may well be regarded as sites of software skills development and organizational know-how that foster open-source production at large, it appears that they are also absorbing, and enhancing the programming abilities of developers who made a start on individual projects, undertaken without formal instruction in their capacity as students.

As can be seen in table 2, the larger projects function as attractors in the overall circulation dynamics of those who change projects. Therefore, on average, agents in second stage projects (with lower private return generated by the public knowledge production) show a higher probability to go to larger FLOSS projects. In particular, this result is valid both for agents that were involved in the first stage in small groups, which are likely to be homogeneous, or in medium groups, which include both homogeneous and heterogeneous cases. The first case corresponds to homogeneous groups that extend to heterogeneous groups to reach the stable coalition size. The second case corresponds to the increase in optimum group size for heterogeneous coalitions, when the private return component of the contribution to the public good is decreasing.

For the agents that were involved in the first stage in large groups, it is more difficult to draw conclusions from the survey as the private learning effects are likely to be less pronounced or at least unequally distributed over the population.

Table 2: Movement of developers from first projects to current/most recent projects

Current/most recent project	First project		
	Small 1-2	Medium 3-29	Large > 29
Large	51	42	32
Medium	30	29	22
Small	13	7	18
Pearson chi-squared (4)	7.96		
Prob>chi-squared	0.09		

Source: David and Shapiro (2008), p. 389, Table 9, Panel B (Data from US-FLOSS 2003 survey, for the sub-set of 244 respondents whose current and first project were not the same)

To develop a model that takes into account the role of these social preferences in coalition formation, the analysis in this paper relies on the general economic approach of intrinsic motivations and applies this general approach to the specific case of group related intrinsic motivations in coalition formation.

In the general economic literature, many studies have shown that individuals can be motivated both by extrinsic motivations (motivations by external rewards such as contingent monetary rewards or rewards from a principal to an agent) or by intrinsic motivations (based on the individuals intrinsic desire to perform the task for its own sake), which leads to contrasted effects on behavior (Benabou and Tirole, 2003). Similarly, in the specific case of public good provision, it has been shown that private contribution to a public good can be motivated by the benefit derived from the aggregate public good or by altruistic motivations (Andreoni, 1988).

To apply these general insights to the specific case where preferences relate

to involvement in group processes, we rely on the well-established empirical literature in social psychology on the role of social motivations in group behavior. As mentioned above, this literature shows that two types of social preferences play a key contribution in the overcoming of social dilemmas that occur in the provision of public goods, which are the group identity effects and the pro-social reputational effects. First, studies provide compelling evidence that the longing for a positive social group identity is a key determinant of engagement in group behavior. Social psychological experiments have shown that social group identity is even in many cases the most important explanatory factor to account for various types of group related motivations such as procedural justice, fairness and supervisor ratings (Blader and Tyler, 2009). Importantly, this research shows that the effects of social group identity depend on group size: social group identity only contributes to overcoming social dilemmas in sufficiently large groups (De Cremer and Leonardelli, 2003). The second type of social preference that plays an important role in group engagement is the social approval for individual pro-social attitudes and behavior (Lepine et al., 2002). These individual pro-social reputational effects have also been studied extensively in the context of the overcoming of social dilemmas (Suurmond et al., 2004; Bolton et al., 2005).

In the case of knowledge goods that jointly generates private/public benefits, the combination of these two effects, the positive social group identity procured by being member of a group that provides a public knowledge good and the social approval for individual pro-social behavior, can provide contrasted effects on coalition formation. Indeed, as we will see below in the development of the model, if the private benefits of the public knowledge good enjoyed by contributors are very substantial, the social approval is likely to decrease as the behavior will be perceived as being based more on self-interest

than on altruistic motives. Therefore, for public knowledge goods generating important private benefits for contributors, social preferences can also inhibit coalition formation, instead of promoting the public good production. In public good theory this will be expressed by the need to reach a larger coalition size (case of difficult coalition formation), compared to coalitions which can already be stable with smaller groups of contributors.

3 Private Production of Public Knowledge Goods: Theory with Standard Preferences

The first step of our analysis builds upon the public good model with coalition formation. The theory of coalition formation, initially developed in context of the negotiation of international environmental agreements (Carraro and Siniscalco 1993, Barret 1994), has been applied to the problem of pure public good provision by Kosfeld et al., (2009). To extend this model to the case of pooled public knowledge goods, this paper draws upon the private-collective incentive theory developed by von Hippel and von Krogh, (2003). More precisely, in line with their extensive case study research, we assume that *contributors to a public good can inherently obtain private benefits that are tied to the development of that good. These benefits are available only to project contributors and not to free riders and represent a form of selective incentives for project participation that need not be managed by collective action project personnel* (von Hippel and von Krogh, 2003, p. 216). This approach differs from the general theory of joint products with standard preferences proposed by Cornes and Sandler (1984; 1994) and further developed by Kotchen (2006, 2009) and Vicary (1997, 2000)[1].

[1]One of the contributions of the work of Cornes and Sandler (1984, 1994) is to show that free-riding over other agents contribution to public goods decreases when goods give

While there exists a large theoretical literature on endogenous coalition formation among countries in the context of global environmental problems (Carraro and Siniscalco 1993, Barret 1994), only few papers consider coalition formation based on the voluntary contribution to public goods by the pooling of endowments held by private actors (Kosfeld et al. 2009, McEvoy 2010). Following these papers we consider a non-cooperative Nash equilibrium situation where agents face a social dilemma and their dominant strategy is to invest their full endowment in the pure private market good, i.e. the numeraire. To make the solution of the game analytically tractable, and to optimize both over the size of the coalition and the level of contribution of the members within the coalition, coalition formation is modeled through a two stage game. During the first stage, the participation stage, agents decide whether or not to participate in the coalition, leading to a certain coalition size. In the second stage, the contribution stage, agents act as joint payoff maximizers and they determine how much of their endowment they will invest in the production of the public knowledge good, knowing that they will receive a shared public benefit from the aggregated public good and a private benefit that is tied to the production of that good. In line with the existing literature, the equilibrium level of participation in the coalition is obtained by solving the game using backward induction, assuming that agents seek to maximize their expected payoff.

We consider an economy populated by i = 1, ..., N individuals each of whom is endowed with exogenous wealth w to be allocated between con-

private benefits to the agents. A stylized example given by Sandler of such an impure public good production model is the case of international development cooperation, where in some arrangements donor countries also derive private benefits from such cooperation, such as through selling of technology, in addition to the global public good benefits related to the overall increase in economic development and poverty alleviation (Sandler and Arce 2007). In this paper, we aim at deriving some basic results from a model that integrates their analysis in a theory of coalition formation with social preferences.

sumption of a pure private good x_i, the numeraire, and a private investment in knowledge g_i to produce a public good G, such that $x_i + pg_i \leq w_i$ and $G = \sum_{i=1}^{N} g_i$. The private investment in knowledge is a voluntary contribution to a public good with a particular characteristic: the contributions, and therefore the private investments, in public knowledge goods are made by agents that are users of that good. In other words, private rewards for contributors are larger than those available to free riders. In this paper we assume that private investments in g_i generate a present private benefit αg_i available only to the contributor that does not depend on the total level of the contribution[2] and a pure public benefit, βG available to all. If for instance g_i is the public source code of open source software, the private benefit αg_i can be derived by technical learning opportunities, personal problem solving or career benefits; if g_i is the contribution to publicly available and certified open databases the private benefit αg_i can be interpreted as the private research benefit for the individual researcher as well as the possibility to attract career attention from colleagues or firms. Moreover, this private return can also be thought of as a small side payment.

The parameter p is defined as the unit cost for contributing to the public knowledge good or the cost of a private investment in innovation. For the purpose of the analysis we assume that w is a monetary endowment and p the monetary unit cost of contribution to a public knowledge good. However, in general the endowment can also be interpreted as available time and p as the cost of one unit of time invested in contributing to the public knowledge

[2]Assuming that private benefits depend on the aggregate level of the contribution G does not change the qualitatively results of the model. It is straightforward to demonstrate that in that particular case coalition formation is more likely because agents can enjoy a higher payoff with respect to the case in which private benefits do not depend on public knowledge production. This latter scenario is the most appropriate one to develop a private-collective model of knowledge production.

good[3].

For the purpose of the analysis we follow Kosfeld et al (2009), in assuming that agent i's material payoff is defined by the linear function $\pi_i = x_i + \alpha g_i + \delta\beta G$, where $\delta \in]0,1[$ models the constant marginal benefit or payoff of the public knowledge good[4]. This analytical formulation of the game leads to knife-edge solutions characterised by full or null private investment in the knowledge good g_i and, therefore, the production or not to the public knowledge good.

In a recent paper, Finus and Rübbelke (2012) study the relationship between public good provision and ancillary benefit in the context of climate change. Assuming quadratic benefit and quadratic cost functions ancillary benefits might raise the attractiveness of participation in international agreements, that is, expand the range of the benefit-cost parameter for which a two player coalition is stable, only if the number of player is sufficiently small. In our private-collective model the private benefits tied to the production of the public knowledge goods play a similar role of ancillary benefits, even if the incentive to free-ride is less pronounced. Indeed, the linear payoff is chosen for the sake of analytical tractability and because the qualitative results of the model do not change significantly using other analytical formulations when the size of the population is sufficiently large (Finus and Rübbelke, 2012).

To remain within the non-cooperative equilibrium scenario, we assume that the technology of g_i is such that the unit cost of contribution p is sufficiently

[3]Notice that the model could also be adapted to apply to coalitions with exclusive knowledge goods that generate public knowledge spillovers, such as certain patent pools: in this case g_i could be defined as the pure marketed good generating public knowledge spillovers. In this case the parameter p has to be interpreted as an exogenous market price, as for instance in the case of intellectual property rights. In this paper we only focus on the more general interpretation of g_i that fits with the private-collective innovation model for open source software developed in von Hippel and von Krogh (2003), that is any form of private investment in generating publicly available knowledge.

[4]In this simple linear case the two goods are substitutes because indifference curves are linear and total utility depends on a linear combination of the quantities of each good consumed.

high compared to the private benefit deriving from the production of the collective good $p > \alpha$[5]. The interaction of the agents generates a n-players prisoner dilemma. We assume a social dilemma scenario, which is guaranteed when $\frac{\bar{\delta}}{N} < \delta < \bar{\delta}$,where $\bar{\delta} = \frac{p-\alpha}{\beta}$ can be interpreted as marginal per capita return cut-off. This assumption implies that the strategy profile $g_i = 0$, $\forall i = 1, .., N$ is the unique non-cooperative Nash equilibrium of the game. Indeed, the game implies that efficient Pareto optimal outcomes involve a full purchasing of the good $g_i = \frac{w}{p}$ because the public characteristic of this particular good plays the same role as a public good in standard theory. Thus, players have an incentive to voluntary create coalitions for pooling the public knowledge good in order to overcome the social dilemma situation[6].

Following the two stage process of general coalition theory, it is now possible to compare the strategy of agents that do not participate in the coalition, to those that participate in the coalition solving the game with backwards induction. First, during the second stage, the contribution stage, it is straightforward to observe that the dominant strategy for each agent not participating in the coalition, labeled with nc, is to purchase only the pure private good. To demonstrate this, first consider the payoff of individuals outside the coalition that respectively contribute to the public knowledge good (g) to those that do not contribute (0):

$$\pi_{g,nc} = w_i - pg_i + \alpha g_i + \delta\beta(g_i + G_{-i}) \tag{1}$$

[5]If the cost of contributing to a public knowledge good is too low, i.e. $p < \alpha$, then the numeraire x_i is never acquired. In this particular case we would have no social dilemma situation: the optimal strategy for all individuals would be to full contribute to the knowledge good because its cost is lower than the marginal benefit given by the private benefits tied to the production of that good.

[6]As our model considers the case of knowledge goods that are available to all, the standard public good game is a particular case of our model when $\alpha = 0$ and $\beta = 1$. Public good production is given by the sum of the non-excludable component of the investment or contribution in knowledge of individual i, βg_i and all other agents in the economy, βG_{-i}.

$$\pi_{0,nc} = w_i + \delta\beta(G_{-i}) \tag{2}$$

Let define $\Delta\pi_{nc} \equiv \pi_{g,nc} - \pi_{0,nc}$. After some algebraical manipulations we obtain:

$$\Delta\pi_{nc} = g_i[\delta\beta + \alpha - p] \tag{3}$$

The optimal individual behaviour in the fringe depends on the sign of the term in brackets in (3). It is straightforward to observe that for each agent outside of the coalition $\Delta\pi_{nc}$ is maximised when $g_i = 0$ if $\delta < \bar{\delta}$ or when $g_i = \frac{w_i}{p}$ if $\delta > \bar{\delta}$. Under social dilemma, the latter condition can never hold. Therefore each member not participating in the coalition will never invest private resources in a knowledge good of such characteristics.

Next, we consider the payoffs of members of a coalition, labeled with c, contributing (g) or not (0) to the knowledge public good. This payoff will be respectively given by:

$$\pi_{g,c} = w_i - pg_c + \alpha g_c + \delta\beta(G_c + G_{nc}) - \theta g_i \tag{4}$$

$$\pi_{0,c} = w_i + \delta\beta(G_{nc}) \tag{5}$$

with βG_c the total public knowledge good produced by the coalition via the private investment of the coalition members, g_c, and βG_{nc} the public knowledge good level produced outside the coalition. Defining with s the size of the coalition we can define $G_c = sg_c$.

As in Kosfled et al., (2009), we introduce a cost of participating in the coalition. In the two-stage game, the cost of participating in the coalition can be supposed to be a constant fraction of the total contribution to the public knowledge good by the agents. Therefore, in presence of identical individuals, the average participation costs coincide with individual cost, θg_c.

After some algebraical manipulations we derive $\Delta\pi_c \equiv \pi_{g,c} - \pi_{0,c}$:

$$\Delta\pi_c = g_c[\delta\beta s + \alpha - p - \theta] \tag{6}$$

The sign of the expression in the brackets in (6) is conditional on the size of the coalition and on the cost of the coalition formation. Given linearity of the payoff, members will invest their total wealth in knowledge good if this sign is positive, otherwise they will invest their endowment only in the pure private commodity (the numeraire).

Turning next to the first stage, the participation stage, each agent has to decide if he joins the coalition or not. Solving equation (6) allows to determine the minimum coalition size that is sufficient so that agents will join the coalition. We define by s^{min} this minimum profitable coalition size. The Nash equilibrium associated to the decision to stay or not in the coalition follows from considering the well-known stability conditions developed in D'Aspremont et al. (1983), largely used in the international environmental agreement literature (Carraro and Siniscalco, 1993, Barret, 1994, Ulph, 2004). More precisely, a coalition is stable if and only if it is both internally stable (no member is better off leaving the coalition) and externally stable (no agent outside of the coalition wants to join it).

Proposition 1 *If individuals are identical, there exists a coalition equilibrium of size s^* defined as the smallest integer such that*

$$s^* \geq s^{min} \equiv \frac{p - \alpha + \theta}{\beta\delta}$$

The sub-game perfect Nash equilibrium of the voluntary coalition game is given by $s^* = s^{min}$. When $s^{min} \geq N$ then a grand coalition with $s^* = N$ forms.

Proof. *Proposition 1 follows from the observation that the only non-trivial internally and externally stable coalition is the smallest profitable coalition of size* $s^{min} \equiv \frac{p-\alpha+\theta}{\beta\delta}$. *Knowing that under social dilemma agents outside the coalition will never contribute to the public knowledge good, we observe:*

$$\pi_c(s^{min}) \equiv w - (p-\alpha)g_c + \delta\beta s^{min} g_c - \theta g_c \geq w \equiv \pi_{nc}(s^{min} - 1)$$

if and only if $s \geq s^{min} \equiv \frac{p-\alpha+\theta}{\beta\delta}$ *However, if the coalition has a size* $s > s^{min}$, *then at least one member has an incentive to defect while the remaining members would continue to stay in the coalition that produces the public knowledge good. Therefore, the only internally stable coalition is the one of size* $s = s^{min}$. *This coalition is also the only externally stable because no agent outside the coalition will find it profitable to join it, that is:*

$$\pi_c(s^{min}+1) \equiv w-(p-\alpha)g_c+\delta\beta(s^{min}+1)g_c-\theta g_c < w+\delta\beta s^{min} g_i \equiv \pi_{nc}(s^{min})$$

for $\delta < \frac{p-\alpha+\theta}{\beta}$. *Considering positive participation cost,* θ, *this latter condition is always verified in social dilemma scenarios where, by hypothesis,* $\delta < \bar{\delta} \equiv \frac{(p-\alpha)}{\beta} < \frac{(p-\alpha)+\theta}{\beta}$.

It is straightforward to demonstrate that the grand coalition $s^* = N$ *is always internally and externally stable. In this case all the agents of the economy will join the coalition during the participation stage of the game.* ■

The participation threshold s^{min} determines the minimum number of individuals required in the participation stage to overcome the social dilemma. In this setting the non trivial internally and externally stable coalition is given by the smallest profitable coalition of size s^{min}. If $s < s^{min}$ coalition members do not contribute to the knowledge good after the participation stage and the

$N - s$ agents that do not participate in the coalition maximize their expected payoffs by not contributing to the knowledge good. Similarly, when $s > s^{min}$ at least one member would find it profitable to leave the coalition and internal stability will not be reached.

The comparative statics shows that the effects of both private, α, and public, β, benefit tied to the public knowledge good production on the optimal coalition size are negative. Interestingly, one can observe an ambiguous effect of a change in α on the aggregate welfare of the coalition equilibrium. The latter can be defined as the sum of the welfare of members, $\Pi_{g,c} = \sum_{i=1}^{s} \pi_i$, and non-members of the coalition, $\Pi_{0,nc} = \sum_{i=s+1}^{N} \pi_i$, of size s. We label the aggregate welfare $\Pi_{coal} = \Pi_{g,c} + \Pi_{0,nc}$. Notice that:

$$\frac{\partial \Pi_{coal}}{\partial \alpha} > 0 \Rightarrow s^* < N < 2s^* \qquad\qquad \frac{\partial \Pi_{coal}}{\partial \beta} > 0 \ \forall \ N > s^*$$

The aggregate welfare under coalition equilibrium is increasing with the private benefit α if the size of the population is not too large. The derivative of Π_{coal} with respect to α is given by $\frac{w[2(p-\alpha+\theta)-\beta\delta N]}{\beta\delta p}$. This derivative is positive if and only if $s^* < N < 2s^*$. When the grand coalition is not formed, that is $s^* < N$, and total population is not too large, $N < 2s^*$, then the reduction in the number of coalition members due to an increase in the private benefit α is dominated by the increased size and aggregate welfare of the agents outside the coalition, $(\Pi_{0,nc})$. When the size of the population is large compared to the coalition size, i.e. $N > 2s^*$, the effect of α on aggregate welfare is negative because the effect of α on the size of the coalition becomes more important. By contrast, the β component always positively impacts the aggregate welfare under coalition equilibrium, via the non-excludable benefits of the knowledge public good.

A specific case that deserves mentioning is the case in which the technology of the knowledge good is such that the private and public benefits act as perfect substitutes, for example when $\alpha + \beta = 1$, so that is not possible to increase α, without decreasing β. In the latter case the effect of the increase in the private component α (and the related decrease in public component β) depends on both the cost of participating in the coalition and the cost of contributing to the knowledge good. If the cost of contribution is sufficiently high compared to the price of the numeraire (normalized to 1), $p > 1 - \theta$, then an increase in the private benefit α will increase the optimal coalition size. However, when the cost of contributing to the knowledge good is sufficiently low, $p < 1 - \theta$, then the coalition size will decrease.

4 Private Production of Public Knowledge Goods: Theory with Social Preferences

This section broadens the model of coalition formation to the case of social preferences, by integrating the evidence from social psychology on the role of social preferences in group behavior. Indeed, in the standard model, adding private benefits to the coalition theory always leads to making coalition formation more easy. This is however contradicted by the empirical evidence of open source communities, amongst others, which shows that private benefits related to social preferences can make coalition formation more difficult in certain circumstances. To differentiate the voluntary pools of knowledge public goods with social preferences from the pools with standard preferences, we designate the former in short by social network coalitions. As above, for analytical tractability, we build a basic linear model that accounts for the main results from the empirical literature.

As stated in section 2, the empirical literature in social psychology highlights two key dimensions of social preferences that play a prominent role in the involvement in group behavior: social group identity related to the collective goals realized by the group and social approval for pro-social attitudes. To model the effect of the positive social group identity related to the belonging to a group that provides public goods, we should consider both the importance of the collective goal related to the aggregate public good provision (cf. for example Wit and Ker, 2002, p. 634) and the fact that group size effects play a role (De Cremer and Leonardelli, 2003). Therefore the social group identity component of the social preference function can be considered as being proportional to the aggregate public good benefit of the knowledge public good produced by the coalition, βG.

For modeling the social approval of the pro-social behavior in public knowledge good production, we suppose that the social approval is proportional to the perceived individual pro-social or altruistic behavior, as measured by the difference between the individuals contribution to the public knowledge good and his private benefit derived from the production of the public good, that is $(\beta - \alpha)g_i$. This means that when $\beta > \alpha$, the function that gives the social approval of the pro-social behavior is defined as being positive, while when $\beta < \alpha$, this function is defined as being negative. The latter correspond to a social disapproval of a behavior that contributes more to the personal private utility (through α) than to the public good (through β).

Depending on the context, decline in one of these dimensions has shown to lead to a more or less steep decline in the provision of collective benefits in organizations (Joireman et al., 2006; Penner and Finkelstein, 1998). Social group identity and individual pro-social reputation can be aggregated into the weighted function: $a_i = (1 - \rho)\beta G + \rho(\beta - \alpha)g_i$, with $0 \leq \rho \leq 1$ defined as

the weight or subjective value that members of a group give to individual pro-social reputation compared to the social group identity. If $\rho \to 1$, then agents do not exhibit social preferences for the building of the social group identity through the aggregate public good production. Rather they care about signaling and approving/disapproving personal pro-social reputations related to making knowledge publicly available, such as in the case of relative disapproval of the community of hackers discussed in section 2. By contrast, when $\rho \to 0$ the agents social preferences are only based on their belonging to a group with a positive social group identity related to the aggregated production of the public knowledge good.

Representative agent's preferences are described by the utility function $u_i = \pi_i + v(a_i)$ where function v is assumed to be linear for the sake of simplicity[7]. We are interested in social dilemma scenarios, which are characterized as above by a situation in which only the pure private commodity (the numeraire) is purchased by agents, but where there exists an incentive for the creation of a coalition where each coalition member invests in the production of the public knowledge good. Individuals maximize utility $u_i = \pi_i + v(a_i)$ taking into account the behavior of other agents and the social consequences of their personal actions when investing resources in the knowledge good. Social dilemma is guaranteed when the marginal material benefit to the non-excludable component is such that $\frac{\bar{\bar{\delta}}}{n} < \delta < \bar{\bar{\delta}}$ with $\bar{\bar{\delta}} = \frac{p-\alpha(1-\rho)-\beta}{\beta}$.

As in the benchmark case, an increase in parameters α and β reduces the marginal per capita cut-off payoff $\bar{\bar{\delta}}$, while ρ impacts positively $\bar{\bar{\delta}}$. The assumption of a non-cooperative Nash equilibrium (the social dilemma scenario) implies that individuals outside the coalition will never contribute to the public knowledge good. Assuming such a social dilemma situation with agents

[7] A similar approach can be found in Hollander (1990) when agents have preferences for social approval conceptualized as an emotional activity.

that have social preferences, we can derive the optimal size of a coalition that is able to overcome the non-cooperative equilibrium by comparing the utility of a member of the coalition contributing to the knowledge good with the utility of a member that prefers not to invest resources in this type of good. Proceeding as in the previous section, we obtain for coalition members[8]:

$$\Delta u_c \equiv u_{g,c} - u_{0,c} = g_c\{s\beta[(1-\rho)+\delta] + \alpha - p + \rho(\beta - \alpha)\} \tag{7}$$

Proposition 2 *When agents exhibit social preferences for social identity and pro-social attitudes, $a_i = (1-\rho)\beta G + \rho(\beta - \alpha)g_i$, the optimal stable coalition is given by:*

$$s^{**} = \frac{p - \alpha + \theta + \rho(\alpha - \beta)}{\beta[(1-\rho)+\delta]}$$

The coalition of size s^{**} is the sub-game perfect Nash equilibrium of the voluntary coalition game with social preferences. When $s^{**} \geq N$ then a grand coalition with $s^{**} = N$ forms.

Proof. *To demonstrate that a coalition of size s^{**} is the sub-game prefect Nash equilibrium of the game with social preferences we proceed as in proposition 1. Solving the term in brackets in (7) we find the minimum profitable coalition size $s^{min} = \frac{p-\alpha+\theta+\rho(\alpha-\beta)}{\beta[(1-\rho)+\delta]}$. Assume a coalition with a number of members equal to the minimum profitable size. We want to prove that if one member of the social network defects then no member will find it convenient to contribute to the public knowledge good. Comparing the utility of an agent when at least one member leaves the coalition $(s^{min} - 1)$ with the utility of a member when the minimum profitable size is reached s^{min} allows to observe that $u_c(s^{min}) \geq u_{nc}(s^{min} - 1)$ if and only if $s \geq s^{min} = \frac{p-\alpha+\theta+\rho(\alpha-\beta)}{\beta[(1-\rho)+\delta]}$. How-*

[8]As in the previous section, it is straightforward to demonstrate that, under social dilemma, agents outside the coalition will never contribute to the public knowledge good.

ever, if the coalition size is strictly greater than s^{min} then at least one member will find it profitable to leave the coalition. Using s^{min} we easily observe that $u_c(s^{min}) = u_{nc}(s^{min} - 1)$. It follows that the only internally stable coalition is the one with size s^{min}.

External stability requires that $u_c(s^{min}+1) < u_{nc}(s^{min})$, meaning that an agent outside the coalition will never find it profitable to join the coalition during the participation stage. Comparing $u_c(s^{min}+1) \equiv w-(p-\alpha)g_c+\delta\beta(s^{min}+1)g_c-\theta g_c+(1-\rho)\beta(s^{min}+1)g_c+\rho(\beta-\alpha)g_c < w+\delta\beta s^{min}g_c+(1-\rho)\beta s^{min}g_c \equiv u_{nc}(s^{min})$ we observe that the coalition s^{min} is externally stable if and only if $\delta < \frac{p-\alpha(1-\rho)-\beta+\theta}{\beta}$. This condition is always satisfied under social dilemma with social preferences, $\delta < \bar{\bar{\delta}}$. It follows that the optimal coalition size s^{**} coincides with the minimum profitable coalition size s^{min}

As before, the grand coalition is the trivial outcome of the game when $s^{**} \geq N$, that is, the entire population will join the coalition during the participation stage of the game. ■

The optimal coalition of size s^{**} coincides with the smallest profitable coalition also for agents with social preferences. However, as the discussion in section 2 of the stylised facts of coalition formation in FLOSS communities shows, the social preferences produce contrasted effects on the facility of coalition formation. Using the model of coalition formation with social preferences, it is possible to show that assuming social preferences indeed does not necessarily imply that the stable coalition size will be smaller than in the case of standard preferences.

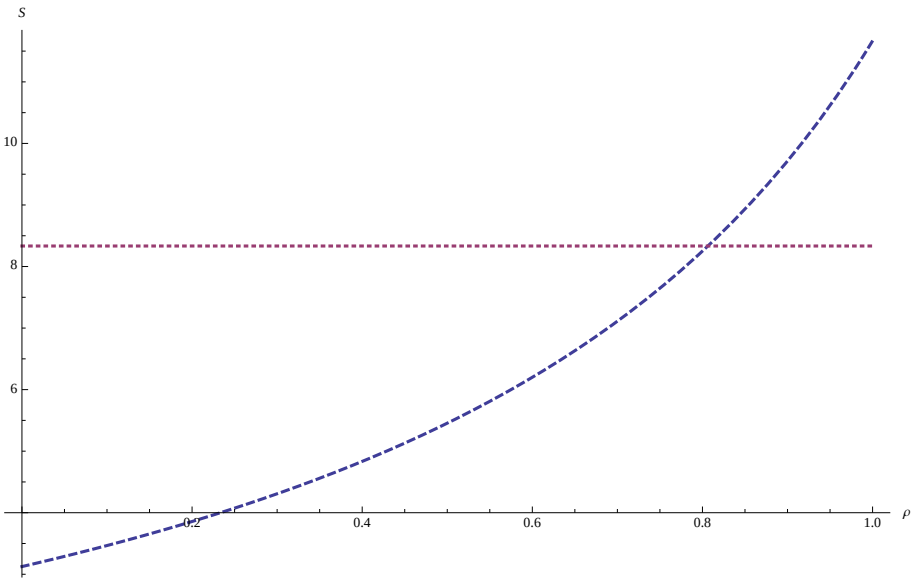
Corollary 1 *If $\beta < \alpha$, depending on weight ρ, the optimal coalition size created by agents with social preferences can be larger or smaller with respect to the optimal size of a coalition formed by individuals with standard preferences:*

- *if* $\rho > \bar{\rho} \Rightarrow s^{**} > s^{*}$
- *if* $\rho < \bar{\rho} \Rightarrow s^{**} < s^{*}$

with $\bar{\rho} = \frac{p-\alpha+\theta}{p-\alpha+\delta(\alpha-\beta)+\theta}$. *If* $\beta \geq \alpha$ *the coalition size under social preferences is always smaller than (or equal to) the coalition size under standard preferences,* $s^{**} \leq s^{*}$.

Comparing the optimal coalition size under social and standard preferences, we observe that s^{**} can be larger of s^{*} if and only if $\rho > \bar{\rho} \equiv \frac{p-\alpha+\theta}{p-\alpha(1-\delta)-\beta\delta+\theta}$. If the public reward to the public knowledge good is sufficiently small, e.g. $\beta < \alpha$, and the weight to pro-social reputation is such that $\rho > \bar{\rho}$, then the coalition formation will be more difficult to form for agents with social preferences compared to the scenario in which agents exhibit standard preferences. By contrast, if the weight for social group identity related to the collective action is sufficiently high, so that $\rho < \bar{\rho}$, then the optimal coalition size for agents with social preferences will be smaller with respect to the coalition composed by agents with standard preferences, $s^{**} < s^{*}$.

Figure 1: Optimal size under standard and social preferences



As stated in the corollary, if the technology of the knowledge good is such that $\beta < \alpha$, then the effects of the social preferences are ambiguous: depending on the weight for group identity or comparative approval, it might or might not make coalition formation easier. This situation is represented in figure 1, which graphically describes the optimal coalition size for different values of the comparative weight given to social approval for pro-social attitudes[9] ρ, compared to the weight given to the social group identity $(1 - \rho)$ when $\beta < \alpha$. Figure 1 illustrates that if the weight for pro-social reputation is sufficiently high, $\rho > 0.785$, then the optimal coalition size for agents with social preferences (dashed curve) will be larger than the optimal size for agents with standard preferences (dotted line), and therefore the coalition will be more difficult to form. The larger the weight for the individual pro-social reputation in the social preferences function compared to the social group identity (e.g. hackers communities), the larger the optimal coalition size under social preferences will be.

As stated above, if the technology of the knowledge good is such that $\beta < \alpha$, depending on the weight for group identity or comparative approval, then the effects of the social preferences is ambiguous: it might or might not make coalition formation easier. The effect of α on the aggregate welfare when a coalition equilibrium is reached is the same as the one observed in the case of standard preferences. As in the benchmark case, the total welfare is increasing in the private benefit α if the size of the population is not too large, that is $s^{**} < N < 2s^{**}$. Otherwise it is negative.

[9] We use the following parameters' values for this numerical example: $\beta = 0.2, \alpha = 0.6, p = 1.5; \theta = 0.1; \delta = 0.6$.

5 Application: Heterogeneous Social Preferences

As highlighted in section 2, the ambiguous effects of the social preferences on coalition formation allows to envision an alternative strategy to overcome social dilemma situations in case of small and medium homogeneous groups. As seen from the empirical studies on FLOSS, groups with homogeneous social preferences and small private benefits can overcome social dilemmas by broadening homogeneous groups to larger heterogeneous groups. To explain this feature, we assume in this section a heterogeneous population in which individuals differ in terms of their social preferences for social group identity and social approval of individual pro-social attitudes. More precisely, we assume two types of agents who differs only in terms of their social preference function: agents exhibit either more weight for individual pro-social reputation $\bar{\rho}$ or they exhibit more weight for social group identity related to the provision of the public knowledge good by the coalition $\underline{\rho}$, with $\bar{\rho} > \underline{\rho}$. To illustrate this situation based on the categories of the FLOSS surveys (cf. table 1), one can think for instance of a population of developers in which some members participating in social networks have strong preferences for their private pro-social reputation, such as in the case of the social disapproval effects of a community of hackers developing open source software in the FLOSS surveys, while other agents exhibit social preferences for building the groups social identity through their involvement in the public good provision, such as social programmers or professionals. The questions that we want to investigate in this context are the following: what is the optimal coalition size when the social networks are composed of agents with different social preference functions? Is the coalition formed by heterogeneous agents stable? Does this type of coalition form when

homogeneous groups are not able to implement a community composed by agents with the same preferences?

Assume that individuals are rational. They anticipate the strategy of other agents when maximizing their utility function in contributing to the knowledge good. The choice of participation is made simultaneously by both type of agents. In order to derive the minimum profitable mixed coalition size when the social preferences are heterogeneous, we have to solve the following system of equations:

$$\begin{cases} \Delta\bar{u} = w_i - [(p-\alpha) - \theta + \beta\delta s^m]g_i + (1-\bar{\rho})\beta s^m g_i + \bar{\rho}(\beta-\alpha)g_i - (w_i + \beta\delta\underline{s}g_i) \\ \Delta\underline{u} = w_i - [(p-\alpha) - \theta + \beta\delta s^m]g_i + (1-\underline{\rho})\beta s^m g_i + \underline{\rho}(\beta-\alpha)g_i - (w_i + \beta\delta\bar{s}g_i) \end{cases} \tag{8}$$

with $\bar{s}$ and $\underline{s}$ defining the number of members respectively with social preferences $\bar{\rho}$ and $\underline{\rho}$ participating in the heterogeneous coalition. The sum $s^m = \bar{s}+\underline{s}$, namely mixed coalition, defines the size of the minimum profitable coalition when agents exhibit heterogeneous social preferences and the choice of participation is made simultaneously. When maximizing the net utilities $\Delta\bar{u}$ and $\Delta\underline{u}$, rational agents know the number of members of the other groups willing to participate in the mixed coalition. Therefore, they also know their personal material benefit of the free riding when not participating.

Solving the system defined in (8) allows to derive the minimum profitable mixed coalition size formed by agents with different social preferences as well as the share of each group within the coalition. We find that:

$$\bar{s} = \frac{\beta(\underline{\rho}-\bar{\rho}) - \alpha\delta(1-\bar{\rho}) + (p+\theta)(\delta+\bar{\rho}-\underline{\rho}) - \beta\delta\bar{\rho}}{\beta\delta(2+\delta-\bar{\rho}-\underline{\rho})} \tag{9}$$

$$\underline{s} = \frac{\beta(\bar{\rho}-\underline{\rho}) - \alpha\delta(1-\underline{\rho}) + (p+\theta)(\delta+\underline{\rho}-\bar{\rho}) - \beta\delta\underline{\rho}}{\beta\delta(2+\delta-\bar{\rho}-\underline{\rho})} \tag{10}$$

$$s^m = \frac{2(p-\alpha+\theta)+(\alpha-\beta)(\underline{\rho}+\bar{\rho})}{\beta(2+\delta-\bar{\rho}-\underline{\rho})} \tag{11}$$

Proposition 3 *The condition $\rho^- < \bar{\rho} \leq \rho^+$ is a necessary but not a sufficient condition to guarantee the stability of the mixed coalition.*

(i) Assume $\alpha > \beta$. If $\rho^- < \bar{\rho} \leq \rho^+$ then the mixed coalition of size s^m, with $\bar{s}$ members of type $\bar{\rho}$ and $\underline{s}$ members of type $\underline{\rho}$, with $\underline{s} < \bar{s}$, is the sub-game Nash equilibrium of the game;

(ii) assume $\beta > \alpha$. If $\underline{\rho} < \frac{p-\alpha+\theta}{\beta-\alpha}$ and $\rho^- < \bar{\rho} \leq \rho^+$ then same result as (i); if $\underline{\rho} \geq \frac{p-\alpha+\theta}{\beta-\alpha}$ then the mixed coalition of size s^m is not stable.

Proof. *Assume that agents are not able to form a coalition with individuals of the same social preferences type, that is $s < s^{**}$ for both $\underline{\rho}$ and $\bar{\rho}$. This implies that when the heterogeneous coalition is not formed the public knowledge good is not produced. Define with $\rho^- = \frac{\alpha\delta+(p+\theta)(\underline{\rho}-\delta)-\beta\underline{\rho}}{p+\delta(\alpha-\beta)+\theta-\beta}$ and $\rho^+ = \frac{(\beta-\alpha)\delta(1-\underline{\rho})}{p+\theta-\beta} + \delta + \underline{\rho}$. Notice that if $\alpha > \beta$ then $\rho^- < \rho^+$. First, assume that $\alpha > \beta$. In order to study the stability of the minimum profitable mixed coalition s^m we have to prove that simultaneously both type of members have no incentive to leave the coalition and that agents outside the coalition have no incentive to join it. Comparing the utility of an agent with social preferences $\underline{\rho}$ when one member leaves the coalition with the utility of a member with social preferences $\underline{\rho}$ when the size s^m is reached, allows to observe that $\underline{u}_c(s^m) - \underline{u}_{nc}(s^m-1) = \frac{g[\beta(\underline{\rho}-\bar{\rho})-\alpha\delta(1-\bar{\rho})+(p+\theta)(\delta+\bar{\rho}-\underline{\rho})-\beta\delta\bar{\rho}]}{\beta\delta(2+\delta-\bar{\rho}-\underline{\rho})} \equiv g\bar{s} \geq 0$ if and only if $\bar{\rho} \geq \rho^-$. If the coalition size is strictly greater than s^m, then at least one member will find it profitable leave the coalition. Using (11) we observe that $\underline{u}_c(s^m) = \underline{u}_{nc}(s^m-1)$ when $\bar{\rho} = \rho^-$. Knowing that $\bar{\rho} > \underline{\rho}$ by assumption, it follows that only when agents exhibit social preferences such that $\bar{\rho} > \rho^-$ the mixed coalition s^m is profitable and internally stable for agents with social preferences $\underline{\rho}$. This coalition is also externally stable for agents with social*

preferences $\underline{\rho}$*. It is easy to prove that the external stability condition is verified, that is,* $\underline{u}_c(s^m+1) < \underline{u}_{nc}(s^m)$ *for all* $0 \leq \underline{\rho} < \bar{\rho}$*.*

Similarly, for an agent with social preferences $\bar{\rho}$*, internal stability is reached if and only if* $\bar{u}_c(s^m) - \bar{u}_{nc}(s^m-1) = \frac{g[\beta(\bar{\rho}-\underline{\rho})-\alpha\delta(1-\underline{\rho})+(p+\theta)(\delta+\underline{\rho}-\bar{\rho})-\beta\delta\underline{\rho}]}{\beta\delta(2+\delta-\bar{\rho}-\underline{\rho})} \equiv g\underline{s} \geq 0$*. This condition is verified when social preferences are such that* $\bar{\rho} \leq \rho^+$*. When this latter condition is met the mixed coalition* s^m *is profitable and internally stable for agents with social preferences* $\bar{\rho}$*. Also for this type of agents the coalition is externally stable,* $\bar{u}_c(s^m+1) < \bar{u}_{nc}(s^m)$ *for all* $\underline{\rho} < \bar{\rho} \leq 1$*. Therefore, the mixed coalition* s^m *is an internally and externally stable coalition and it is the sub-game Nash equilibrium of the game if and only if social preferences are such that* $\rho^- < \bar{\rho} \leq \rho^+$*.*

Assume now that $\beta > \alpha$*. In this case it is possible that the mixed coalition is not stable also when* $\rho^- < \bar{\rho} \leq \rho^+$*. If the preferences of the agent* $\underline{\rho}$ *are such that* $\underline{\rho} \geq \frac{p-\alpha+\theta}{\beta-\alpha}$*, than* $\rho^- \geq \rho^+$ *and internal stability is never reached. By contrast, if* $\underline{\rho} < \frac{p-\alpha+\theta}{\beta-\alpha}$ *and* $\rho^- < \bar{\rho} \leq \rho^+$ *the mixed coalition is internally and externally stable, for the reasons discussed above.* ■

In the model with heterogeneous population an increase (decrease) in the private benefit tied to the public knowledge good α also leads to smaller (bigger) optimal size s^m of the mixed coalition. In other words, an increase (decrease) in private return generated by the public knowledge good production makes coalition formation easier (more difficult). For example, in the FLOSS case discussed in section 2, for a heterogeneous coalition composed of social programmers (with a comparative high social preference for social group identity, that is a low $\underline{\rho}$) and hackers (with a comparative high social preference for the approval of individual pro-social behavior related to freeing the knowledge, that is a high $\bar{\rho}$), this means that a decrease in private return will lead to

bigger coalition sizes. This can be explained by two contrasting effects: the decrease in private return leads to a lower willingness to participate in small coalitions by the social programmers, but the same decrease leads to a higher willingness to participate in small coalitions by the hackers. However, the social group identity effect dominates the impact on the individual reputation in the heterogeneous coalition.

Interestingly enough, the model shows that when a social network with members characterized by the same type of preferences is not formed because the minimum profitable coalition size is not reached, a mixed coalition with both type of members might be implemented. Imagine for example that in the participation stage agents of type j, with $j \in \{\underline{\rho}; \bar{\rho}\}$ and s^{**} the optimum coalition size of the homogeneous coalition, do not form any coalition because $s^j < s^{**}$. However, it is possible that a mixed coalition $s^m = \bar{s} + \underline{s}$ is implemented and is stable even when $s^j = \bar{s}$ for $j = \bar{\rho}$ and $s^j = \underline{s}$ for $j = \underline{\rho}$. This result is driven by the fact that agents might find it profitable to collaborate with agents with different social preferences in order to reach the minimum profitable coalition size that they are not able to reach when creating a community of individuals with the same social preferences[10].

A numerical example can help to understand this statement. Assume the following parameter values: $p = 1.5$, $\delta = 0.3$, $\alpha = 0.8$, $\beta = 0.3$, $\theta = 0.05$ and $\underline{\rho} = 0.5$. If a homogeneous coalition is not formed because $s^j < s^{**}$ a mixed coalition can be formed and can be stable if the condition $\rho^- < \bar{\rho} \leq \rho^+$ is satisfied, that is $0.28 < \bar{\rho} < 0.74$. Table 3 show the approximated minimum profitable size required for the formation of both, the homogeneous and the mixed coalitions, for different level of $\bar{\rho} > \underline{\rho}$.

[10] As can be seen from proposition 3 (ii), this outcome, to be valid in general, requires the presence of agents with strong social preferences for social group identity related to the public good production by the coalition (that is low ρ).

Table 3: Minimum Profitable Size: Homogeneous and Mixed Coalitions

$\bar{\rho}$	s^{**} (homogenous $\underline{\rho}$)	s^{**} (homogeneous $\bar{\rho}$)	$\underline{s}$ (mixed)	$\bar{s}$ (mixed)	s^m
0.55	4	5	2	3	5
0.60	4	5	2	4	6
0.65	4	6	1	5	6
0.70	4	6	0	6	6
0.74	4	7	0	7	7

From Table 3 we observe that when $\bar{\rho} > \underline{\rho} = 0.5$ mixed coalitions are formed by a majority of agents of type $\bar{\rho}$. This result is in line with Proposition 2, showing that individuals with strong preferences for pro-social reputation form larger communities. When $\bar{\rho}$ increases with respect $\underline{\rho}$ agents with preferences for social group identity are less willing to form a social network with agents with strong preferences for pro-social reputation. If for instance $\bar{\rho} = 0.6$ and both group are not able to form a homogeneous coalition of optimal coalition size respectively of 4 and 5 (for example because they have respectively only a group size of 3 and 4), then a mixed coalition composed by 2 agents of type $\underline{\rho}$ and 4 agents of type $\bar{\rho}$ can be formed and profitable. When $\bar{\rho} \to \rho^+$ then the social network or mixed coalition is not anymore formed and individuals with preferences $\underline{\rho}$ do not participate in the social network. In this extreme case the mixed coalition coincides with the homogeneous coalition of agents of type $\bar{\rho}$. In other words, the larger the distance between the weight for pro-social reputation in the two groups, the less heterogeneous the social network will be. Overall these results show a theoretical way to account for the fact that in some cases heterogeneity has a positive impact on collective action, but that in other cases the presence of agents with very diverse social preferences can prevent collaboration and, therefore, the formation of mixed coalitions.

Finally, also when the private return of the public knowledge good is relatively low compared to its public benefit, $\alpha < \beta$, it is possible that mixed

social networks fail to form (second part of proposition 3 (ii)). In particular, if no agent has a strong preference for social group identity or, in other words, if the two groups of agents have very similar and high preferences for individual pro-social reputation, then the mixed community might not be stable. Assume for instance that $\underline{\rho}$ is sufficiently high: $\underline{\rho} > \frac{p-\alpha+\theta}{\beta-\alpha}$. This implies that both social groups have strong social preferences for pro-social attitudes, that is they have strong disapproval of self-regarding attitudes (imagine for instance two groups of hackers with similar preferences in a context in which the private return to public good production is low). If these conditions are met the mixed coalition is not an equilibrium of the game. In this case agents form an optimal coalition only with individuals of the same type.

6 Conclusion

This paper has analyzed the ambiguity of private benefits in fostering coalition formation with publicly available knowledge goods. Such private benefits tied to public goods are considered as an important driver for the proliferation of pooled knowledge goods in social networks. Private benefits to contributors that have been widely studied in the literature are of two kinds: (1) direct private benefits such as individual problem solving or higher citation rates for researchers; (2) satisfaction of social preferences such as group belonging, group identity, pro-social individual reputation and status.

In the current literature on coalition formation for public good provision, the effect of these two kinds of private benefits tied in knowledge production is mostly considered to be positive on the facility to form the coalition. No theoretical explanation of the ambiguous effects of private benefits for contributors to public good − that is, in some cases, the private benefits make

the coalition formation easier and in some case more difficult − is provided in the literature. To build a more general model, this paper integrated the theory of public goods and a social psychological model of group related social preferences into coalition theory. This allowed us to show the contrasted effects of social group identity and social approval/disapproval of individual pro-social attitudes on the coalition formation. The presence of agents giving high value to their individual pro-social reputation within a social network can make coalition formation in some situations more difficult and less likely.

Acknowledgments

We gratefully acknowledge co-funding from the European Commission, under the contract of three PF7-projects, including MICRO B3 (Grant agreement 28758), BIOMOT (grant agreement 282625) and GENCOMMONS (ERC grant agreement 284) and co-funding from the National Science Foundation (MIS Global Science Commons).

References

[1] Andreoni, J., 1988. Privately provided public goods in a large economy: the limits of altruism. Journal of Public Economics 35(1), 57-73.

[2] Barret, S., 1994. Self-enforcing international environmental agreements. Oxford Economic Papers 46: 878-894.

[3] Benabou, R. and Tirole, J., 2003. Intrinsic and extrinsic motivation. Review of Economic Studies 70(3): 489-520.

[4] Benkler, Y., 2006. The Wealth of Networks. Yale UP.

[5] Bergstrom, T. Blume, L. and Varian, H., 1986. On the private provision of public goods. Journal of Public Economics 29: 25-49.

[6] Blader, S. L and Tyler, T. R., 2009. Testing and extending the group engagement model: linkages between social identity, procedural justice, economic outcomes, and extrarole behaviour. Journal of Applied Psychology 94(2): 445464.

[7] Bolton, G. E., Katok, E. and Ockenfels, A., 2005. Cooperation among strangers with limited information about reputation. Journal of Public Economics 89(8): 1457-1468.

[8] Carraro, C. and Siniscalco, D., 1993. Strategies for the international protection of the environment. Journal of Public Economics 52(3): 309-328.

[9] Cornes, R. and Sandler, T., 1984. Easy Riders, Joint Production, and Public Goods. The Economic Journal 94(3): 580-598.

[10] Cornes, R. and Sandler, T., 1994 The Comparative Statics Properties of the Impure Public Good Model. Journal of Public Economics 54(3): 403-421.

[11] d'Aspremont, C., Jacquemin, A., and Gabszeiwicz, J. J., 1983. On the Stability of Collusive Price Leadership. Canadian Journal of Economics (16)1: 17-25.

[12] David, P. A., 2005. Koyaanisqatsi in Cyberspace: The Economics of an Out-of-Balance Regime of Private Property Rights in Data and Information. In International Public Goods and Transfer of Technology under a Globalized Intellectual Property Regime, ed. K. E. Maskus and J. H. Reichman, 79-120. Cambridge University Press.

[13] David, P. A., 2008. The historical origins of 'Open Science': An essay on patronage, reputation and common agency contracting in the scientific revolution. Capitalism and Society 3(2): 5.

[14] David, P. A. and Shapiro, J. S., 2008. Community-based production of open-source software: what do we know about the developers who participate ? Information Economics and Policy 20(4): 364-398.

[15] De Cremer, D. and Leonardelli, G.J., 2003. Cooperation in social dilemmas and the need to belong: the moderating effect of group size. Group Dynamics: Theory, Research, and Practice 7(2): 168174.

[16] Finus, M. and Rübbelke, D.T.G., 2012. Public good provision and ancillary benefits: the case of climate agreements, Environmental and Resource Economics.

[17] Giddens, A., 1999. Runaway World: How Globalization is Reshaping Our Lives. London: Profile.

[18] Hess, C. and Ostrom, E., 2007. Understanding Knowledge as a Commons. MIT Press.

[19] Johnson, J. P., 2002. Open source software: private provision of a public good. Journal of Economics and Management Strategy, 11(4), 637-622.

[20] Joireman, J. et al., 2006. Good citizens to the end? It depends: empathy and concern with future consequences moderate the impact of a short-term time horizon on organizational citizenship behaviors. Journal of Applied Psychology 91(6): 1307-1320.

[21] Kosfeld, M. et al., 2009. Institution Formation in Public Goods Games. American Economic Review 99(4): 1135-1355.

[22] Kotchen, M. J., 2006. Green Markets and Private Provision of Public Goods. Journal of Political Economy 114(4): 816-834.

[23] Kotchen, M. J., 2009. Voluntary Provision of Public Goods for Bads: A Theory of Environmental Osets. The Economic Journal 119: 883-899.

[24] Lepine, J. A. et al., 2002. The nature and dimensionality of organizational citizenship behavior: a critical review and meta-analysis. Journal of Applied Psychology, 87(1):52-65.

[25] Lessig, L., 2001. The Future of Ideas. Random House.

[26] Lessig, L., 2008. Remix: Making Art and Commerce Thrive in the Hybrid Economy. Penguin Press.

[27] Penner, L. A. and Finkelstein, M.A., 1998. Dispositional and structural determinants of volunteerism. Journal of Personality and Social Psychology 74(2): 525-537.

[28] Raymond, E. S., 1999. The Cathedral and the Bazaar: Misings on Linx and Open Source by an Accidental Revolutionary. O'Reilly, Sebastopol, CA.

[29] Reichman, J. H. and Uhlir, P.F., 2003. A contractually reconstructed research commons for scientic data in a highly protectionist intellectual property environment. Law and Contemporary Problems 66: 315.

[30] Sandler, T. and D. G. Arce., 2007. New face of development assistance: public goods and changing ethics. Journal of International Development 19(4):527-544.

[31] Suurmond, G., Swank, O.H. and Visser, B., 2004. On the bad reputation of reputational concerns. Journal of Public Economics, 88(12), 2817-2838.

[32] Vicary, S., 1997. Joint production and the private provision of public goods. Journal of Public Economics 63(3), 429-445.

[33] Vicary, S., 2000. Donations to a public good in large economy. European Economic Review 44(3): 609-618.

[34] von Hippel, E. and von Krogh, G., 2003. Open source software and the 'private-collective' innovation model: issues for organization science. Organization science 14 (2), 209-223.

[35] Wit, A. P and Ker, N. L., 2002. Me versus just us versus us all categorization and cooperation in nested social dilemmas. Journal of Personality and Social Psychology 83(3): 616-637.

Institut de Recherches Économiques et Sociales
Université catholique de Louvain

Place Montesquieu, 3
1348 Louvain-la-Neuve, Belgique

ISSN 1379-244X D/2013/3082/20